These results will be applied for designing practical noise filters.

V. Conclusions

The power transmission coefficient for an optical wave beam with a Gaussian field distribution through a system of two aperture stops is obtained by using the beam-mode expansion method which has been used to know the diffraction effects of an aperture.

The optimum conditions that maximize the power transmission coefficient of a fundamental beam mode are also obtained. These conditions coincide formally with those given by Kogelnik and Yariv for the incident wave having a prolate spheroidal-wave function distribution. The maximum power transmission coefficient can be represented as a function of only the acceptance factor.

When the noise originated from the spontaneous emission is added to the incident Gaussian wave beam, it is important to obtain the maximum signal-to-noise ratio at the output. This problem could be solved by the method developed here.

The analysis adopted here can be applied to both circular and square geometries, which is one of the characteristics of the beam-mode expansion method.

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Analysis of Electromagnetic-Wave Modes in Anisotropic Slab Waveguide

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***Abstract*—Electromagnetic-wave modes propagating in anisotropic slab waveguide are analyzed theoretically in detail. The propagation conditions are derived under which waves can propagate along the axis of the guide. A two-dimensional three-layered waveguiding structure consisting of an anisotropic dielectric slab coated on, or immersed in, isotropic surrounding substrate materials is considered as a typical configuration of the guide. Field-intensity distributions of the propagating modes and their propagation constants are obtained by numerical computations. Techniques for achieving the mode discrimination and the single-mode operation are given. Some possible applications in integrated optics are suggested.**

I. Introduction

THE electromagnetic-wave modes propagating along a slab waveguide consisting of isotropic materials have been investigated extensively as a typical boundary value problem of electromagnetic-wave theory. For the last few years, this problem has evoked much interest in connection with the development of optical integrated circuits.

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On the other hand, the analysis of wave modes in a slab waveguide with anisotropic materials is also of great interest from both the theoretical and practical points of view. To the authors' knowledge, however, little work has been done so far on slab waveguides consisting of anisotropic media, and most of it was restricted to the guide using magnetized gyrotropic ferrites [1].

Recently, Wang *et al.* [2] mentioned the possibility of forming optical devices such as gyrators, optical switches, light modulators, etc., using thin-film waveguide with anisotropic materials as substrates. Nelson and McKenna [3] treated the electromagnetic modes of anisotropic dielectric waveguides at p-n junctions, and Andrews [4] discussed the crystal symmetry effects on nonlinear optical processes in optical waveguides. Nevertheless, all these studies reported previously are limited in only special cases, and a satisfactory general treatment of the modes in an anisotropic slab waveguide has not yet been accomplished.

In the present paper, the wave modes in an anisotropic slab waveguide coated on, or embedded in, isotropic substrate materials are analyzed theoretically in detail. We start with a general discussion of the wave propagation in an arbitrary anisotropic material, derive the dispersion relation, and find the propagation conditions under which waves can propagate along the axial direction of the guide.

As a typical model of an anisotropic slab waveguide, a two-dimensional three-layered waveguiding structure consisting of an anisotropic dielectric slab and surrounding isotropic media are considered. The field distributions and the propagation constants of typical modes are obtained by numerical computations.

A class of this type of waveguide offers an additional variety of schemes in integrated optics which is currently a subject of widespread interest.

II. Electromagnetic Fields in Anisotropic Media

To begin with, we consider wave propagation in arbitrary anisotropic media, and derive the conditions to maintain waves which can propagate in the specific direction, the axial direction of the guide.

Anisotropic materials are characterized by tensor permittivity and tensor permeability, designated as $\hat{\epsilon}$ and $\hat{\mu}$, respectively. In the present paper, all the materials involved are assumed loss free. Both $\hat{\epsilon}$ and $\hat{\mu}$ are then Hermitian. Physically realizable fields in any charge- and current-free region of anisotropic medium are governed by Maxwell's equations

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} \quad (1)$$

together with the constitutive relations

$$\boldsymbol{D} = \hat{\epsilon}\boldsymbol{E}$$

$$\boldsymbol{B} = \hat{\mu}\boldsymbol{H}$$

or, in a Cartesian system of coordinates

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz}^* \\ \epsilon_{xy}^* & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{xz} & \epsilon_{yz}^* & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz}^* \\ \mu_{xy}^* & \mu_{yy} & \mu_{yz} \\ \mu_{xz} & \mu_{yz}^* & \mu_{zz} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} \quad (3)$$

where the asterisk stands for the complex conjugate. Since Maxwell's equations (1) are linear partial-differential equations with constant coefficients, the field of waves propagating in the z direction can be represented in terms of the linear combination of elementary plane waves as follows [5]:

$$\text{field} \propto \{\sum_{\beta_x}\sum_{\beta_y} A(\beta_x, \beta_y) \exp[-j(\beta_x x + \beta_y y)]\} \cdot \exp(-j\beta_z z) e^{j\omega t} \quad (4)$$

where $\exp(j\omega t)$ represents sinusoidal time dependence of the field with angular frequency ω, and β_x, β_y, β_z are x, y, and z components, respectively, of the wave vector associated with elementary plane wave

$$\exp[-j(\beta_x x + \beta_y y + \beta_z z - \omega t)]$$

which may yield a field of waves propagating in the specific direction, the z direction. $A(\beta_x,\beta_y)$, a function of β_x and β_y, is an amplitude of each elementary plane wave. Thus the space and time derivatives in (1) become

$$\frac{\partial}{\partial x} = -j\beta_x \qquad \frac{\partial}{\partial y} = -j\beta_y \qquad \frac{\partial}{\partial z} = -j\beta_z \qquad \frac{\partial}{\partial t} = j\omega. \quad (5)$$

Substitution of foregoing relations into (1), jointly with constitutive relations (2) and (3), leads to the following homogeneous simultaneous linear algebraic equations:

$$\begin{pmatrix} \omega\epsilon_{xx} & \omega\epsilon_{xy} & \omega\epsilon_{xz}^* & 0 & -\beta_z & \beta_y \\ \omega\epsilon_{xy}^* & \omega\epsilon_{yy} & \omega\epsilon_{yz} & \beta_z & 0 & -\beta_x \\ \omega\epsilon_{xz} & \omega\epsilon_{yz}^* & \omega\epsilon_{zz} & -\beta_y & \beta_x & 0 \\ 0 & \beta_z & -\beta_y & \omega\mu_{xx} & \omega\mu_{xy} & \omega\mu_{xz}^* \\ -\beta_z & 0 & \beta_x & \omega\mu_{xy}^* & \omega\mu_{yy} & \omega\mu_{yz} \\ \beta_y & -\beta_x & 0 & \omega\mu_{xz} & \omega\mu_{yz}^* & \omega\mu_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \\ H_x \\ H_y \\ H_z \end{pmatrix} = 0. \quad (6)$$

From the condition for a nontrivial solution of the foregoing equations, the following dispersion relation is obtained:

$$\{(a_1\beta_x^4 + a_2\beta_x^3\beta_y + a_3\beta_x^2\beta_y^2 + a_4\beta_x\beta_y^3 + a_5\beta_y^4) + (a_6\beta_x^2 + a_7\beta_x\beta_y + a_8\beta_y^2) + a_9\} + \{(b_1\beta_x^3 + b_2\beta_x^2\beta_y + b_3\beta_x\beta_y^2 + b_4\beta_y^3) + (b_5\beta_x + b_6\beta_y)\} = 0 \quad (7)$$

where the coefficients a_i and b_i are the constants determined by β_z, ω, and tensor components of $\hat{\epsilon}$ and $\hat{\mu}$.

In order that field expression (4) represents the wave traveling along the z direction, the elementary plane waves must consist of a pair of complementary wave families

$$\{\exp[-j(\beta_x x + \beta_y y)]\} \exp[-j(\beta_z z - \omega t)]$$

and

$$\{\exp[-j(\beta_x x + \beta_y y)]\}^* \exp[-j(\beta_z z - \omega t)]$$

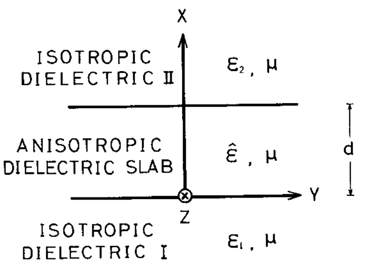

Fig. 1. Cross-sectional geometry of an anisotropic slab waveguide. Guide axis is in the z direction.

giving rise thereby to the wave whose phase velocity coincides with the z axis. In other words, a solution (β_x,β_y) to the dispersion relation (7) must be accompanied by its complex conjugate solution $(-\beta_x^*, -\beta_y^*)$ which also satisfies the same relation (7). It is equivalent to say that the quantity in the brackets { } in (4) must be a real quantity. In view of the above requirement, it can be concluded that the tensor components must fulfill the following conditions, say propagation conditions, to maintain the waves which can propagate in the z direction

$$\epsilon_{yz} = \epsilon_{xz} = \mu_{yz} = \mu_{xz} = 0 \tag{8}$$

or

$$\begin{aligned}\mathrm{Re}\,(\epsilon_{yz}) &= \mathrm{Re}\,(\epsilon_{xz}) = \mathrm{Im}\,(\epsilon_{xy}) \\ &= \mathrm{Re}\,(\mu_{yz}) = \mathrm{Re}\,(\mu_{xz}) = \mathrm{Im}\,(\mu_{xy}) = 0.\end{aligned} \tag{9}$$

Only when the condition (8) or (9) is satisfied, anisotropic materials are capable of transmitting the waves which propagate in the z direction.

III. Electromagnetic-Wave Modes in Anisotropic Dielectric Slab Waveguide

As a typical configuration of an anisotropic slab waveguide, let us consider a two-dimensional three-layered waveguiding structure consisting of an anisotropic dielectric slab and surrounding isotropic substrate materials. Cross-sectional geometry of the guide is shown in Fig. 1, where the axis of the guide is in the z direction. Regions I and II are isotropic dielectric media, whose permittivity and permeability are given by scalar constants, ϵ_1,μ and ϵ_2,μ, respectively, whereas the dielectric layer inserted between them is anisotropic and characterized by tensor permittivity $\hat{\epsilon}$ and scalar permeability μ. Because of the propagation conditions (8), the permittivity tensor $\hat{\epsilon}$ must be of the form

$$\hat{\epsilon} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{xy} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \tag{10}$$

where the tensor components are assumed to be real quantities.[1]

[1] The special case where the permittivity tensor has diagonal terms only will be discussed in the Appendix.

Since the guide is uniform in the y direction, the field is independent of y, and $\beta_y = 0$. Therefore, (6) becomes

$$\begin{pmatrix} \omega\epsilon_{xx} & \omega\epsilon_{xy} & 0 & 0 & -\beta_z & 0 \\ \omega\epsilon_{xy} & \omega\epsilon_{yy} & 0 & \beta_z & 0 & -\beta_x \\ 0 & 0 & \omega\epsilon_{zz} & 0 & \beta_x & 0 \\ 0 & \beta_z & 0 & \omega\mu & 0 & 0 \\ -\beta_z & 0 & \beta_x & 0 & \omega\mu & 0 \\ 0 & -\beta_x & 0 & 0 & 0 & \omega\mu \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \\ H_x \\ H_y \\ H_z \end{pmatrix} = 0. \tag{11}$$

From (11) the dispersion relation can be obtained as follows:

$$\begin{aligned}(k_{xx}^2k_{zz}^2 - k_{xx}^2\beta_x^2 - k_{zz}^2\beta_z^2)\,(k_{yy}^2 - \beta_x^2 - \beta_z^2)& \\ - k_{xy}^4(k_{zz}^2 - \beta_x^2) &= 0\end{aligned} \tag{12}$$

where $k_{xx}^2 = \omega^2\epsilon_{xx}\mu$, $k_{xy}^2 = \omega^2\epsilon_{xy}\mu$, etc. Equation (11), six homogeneous simultaneous linear equations, is subject to restrictive condition (12). It turns out that any five of six equations in (11), together with condition (12), are sufficient for our purpose. Thus we can eliminate any one of the field components, H_z for instance, from (11). All other remaining field components are then expressed by H_z. Eliminating the last equation, (11) can be rewritten in the form

$$\begin{pmatrix} \omega\epsilon_{xx} & \omega\epsilon_{xy} & 0 & 0 & -\beta_z \\ \omega\epsilon_{xy} & \omega\epsilon_{yy} & 0 & \beta_z & 0 \\ 0 & 0 & \omega\epsilon_{zz} & 0 & \beta_x \\ 0 & \beta_z & 0 & \omega\mu & 0 \\ -\beta_z & 0 & \beta_x & 0 & \omega\mu \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \\ H_x \\ H_y \end{pmatrix} = \begin{pmatrix} 0 \\ \beta_x \\ 0 \\ 0 \\ 0 \end{pmatrix} H_z. \tag{13}$$

H_z can be represented by a linear combination of elementary plane waves, each of which is associated with a solution of the dispersion relation (12):

$$H_z = \sum_{n=1}^{2} \{P_n \exp\,(-j\beta_{x_n}x) + Q_n \exp\,(j\beta_{x_n}x)\} \tag{14}$$

where β_{x_n} and $-\beta_{x_n}$ $(n = 1,2)$ are the solution to (12). Here and in the rest of the paper, the term $\exp\,(-j\beta_z z)\exp\,(j\omega t)$ will be omitted for simplicity. Equation (14) may also be expressed in the alternative form

$$H_z = \sum_{n=1}^{2} \{A_n \cos\beta_{x_n}x + B_n \sin\beta_{x_n}x\} \tag{15}$$

in which we have defined

$$\begin{aligned} A_n &= P_n + Q_n \\ B_n &= j(-P_n + Q_n). \end{aligned} \tag{16}$$

A_n and B_n are constants that will be determined by boundary conditions. Substitution of (14) into (13), using A_n and B_n defined by (16), yields the remaining field components. The results are

$$E_z = \frac{\omega\mu}{\beta_z} \sum_{n=1}^{2} a_n\{A_n \cos \beta_{x_n}x + B_n \sin \beta_{x_n}x\}$$

$$H_y = \sum_{n=1}^{2} b_n\{ jA_n \sin \beta_{x_n}x - jB_n \cos \beta_{x_n}x\}$$

$$E_y = -\frac{\omega\mu}{\beta_z} \sum_{n=1}^{2} c_n\{ jA_n \sin \beta_{x_n}x - jB_n \cos \beta_{x_n}x\} \quad (17)$$

where

$$a_n = \frac{k_{xy}^2\beta_z^2}{k_{xx}^2k_{zz}^2 - k_{xx}^2\beta_{x_n}^2 - k_{zz}^2\beta_z^2}$$

$$b_n = \frac{k_{xy}^2k_{zz}^2}{k_{xx}^2k_{zz}^2 - k_{xx}^2\beta_{x_n}^2 - k_{zz}^2\beta_z^2} \cdot \frac{\beta_z}{\beta_{x_n}}$$

$$c = \frac{\beta_z}{\beta_{x_n}}.$$

As we can see from (15) and (17), both H_z and E_z do not generally vanish, and hence the wave modes are said to be hybrid.

On the other hand, the fields in isotropic dielectric substrate, region I, are expressed in the conventional form

$$H_z = C \exp(\alpha_1 x)$$

$$E_z = \frac{\omega\mu}{\beta_z} D \exp(\alpha_1 x)$$

$$H_y = -jpD \exp(\alpha_1 x)$$

$$E_y = j\frac{\omega\mu}{\beta_z} qC \exp(\alpha_1 x) \quad (18)$$

where

$$p = -\frac{k_1^2}{\alpha_1\beta_z} \qquad q = -\frac{\beta_z}{\alpha_1}$$

$$\alpha_1 = (\beta_z^2 - k_1^2)^{1/2} > 0, \qquad k_1^2 = \omega^2\epsilon_1\mu$$

and C and D are constants determined by boundary conditions. Similarly, the fields in isotropic dielectric II are given by

$$H_z = E \exp(-\alpha_2 x)$$

$$E_z = \frac{\omega\mu}{\beta_z} F \exp(-\alpha_2 x)$$

$$H_y = -jrF \exp(-\alpha_2 x)$$

$$E_y = j\frac{\omega\mu}{\beta_z} sE \exp(-\alpha_2 x) \quad (19)$$

where

$$r = \frac{k_2^2}{\alpha_2\beta_z}, \qquad s = \frac{\beta_z}{\alpha_2},$$

$$\alpha_2 = (\beta_z^2 - k_2^2)^{1/2} > 0, \qquad k_2^2 = \omega^2\epsilon_2\mu$$

and E and F are constants determined by boundary conditions.

The x components of the field in three regions, in anisotropic dielectric slab and isotropic dielectric media I and II, can be obtained by the following relations:

$$D_x = \frac{\beta_z}{\omega} H_y \quad (20)$$

$$B_x = -\frac{\beta_z}{\omega} E_y. \quad (21)$$

All the tangential components of the field must be continuous across the boundary surfaces $x = 0$ and $x = d$. Applying these boundary conditions, the coefficients A_1, A_2, B_1, B_2, C, D, E, and F appearing in the foregoing field expressions are determined. The characteristic equation is also derived by applying the boundary conditions as follows:

$$\begin{aligned}
&\sin \beta_{x_1}d \cdot \sin \beta_{x_2}d\{(b_1c_2 - b_2c_1)^2 + (a_1 - a_2)^2pqrs \\
&\quad - (a_1b_2c_2 + a_2b_1c_1)(ps + qr) + (b_1^2 + b_2^2)qs \\
&\quad + (a_1^2c_2^2 + a_2^2c_1^2)pr\} + \cos \beta_{x_1}d \cdot \cos \beta_{x_2}d \\
&\quad \cdot\{(a_1 - a_2)(b_1c_2 - b_2c_1)(pq + rs) - (a_1b_1c_2 + a_2b_2c_1) \\
&\quad \cdot(ps + qr) + 2(a_1a_2c_1c_2pr + b_1b_2qs)\} + \cos \beta_{x_1}d \\
&\quad \cdot\sin \beta_{x_2}d[(b_1c_2 - b_2c_1)\{a_1c_2(r - p) + b_2(q - s)\} \\
&\quad + (a_1 - a_2)\{a_2c_1pr(q - s) + b_1qs(r - p)\}] \\
&\quad + \sin \beta_{x_1}d \cdot \cos \beta_{x_2}d[(b_1c_2 - b_2c_1)\{- a_2c_1(r - p) \\
&\quad - b_1(q - s)\} + (a_1 - a_2)\{-a_1c_2pr(q - s) \\
&\quad - b_2qs(r - p)\}] + (a_1b_2c_1 + a_2b_1c_2)(ps + qr) \\
&\quad - 2(a_1a_2c_1c_2pr + b_1b_2qs) = 0.
\end{aligned}$$

IV. Numerical Examples and Discussion

According to the theoretical analysis described in the preceding section, let us illustrate numerically the field-intensity distributions and the propagation constants of the wave modes propagating in an anisotropic dielectric slab waveguide. The waveguide structure is as shown in Fig. 1, and the tensor permittivity of an anisotropic dielectric slab is assumed to be of the form given by (10).

As an example of anisotropic material, let us consider the electrooptical crystal whose permittivity tensor is given by

$$\hat{\epsilon} = \begin{bmatrix} \epsilon_{\xi\xi} & 0 & 0 \\ 0 & \epsilon_{\eta\eta} & 0 \\ 0 & 0 & \epsilon_{\zeta\zeta} \end{bmatrix} \quad (22)$$

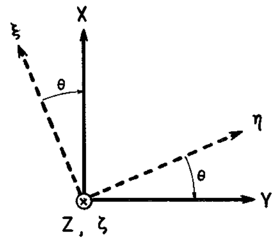

Fig. 2. System of coordinates (x,y,z) used for the analysis and the electrical principal axes (ξ,η,ζ).

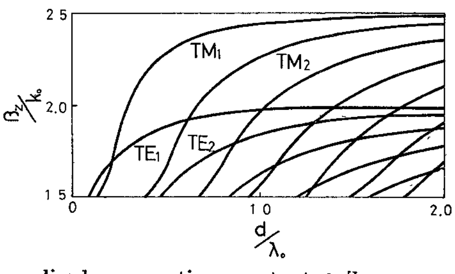

Fig. 3. Normalized propagation constant β_z/k_0 versus normalized frequency d/λ_0 with $\theta = 0°$.

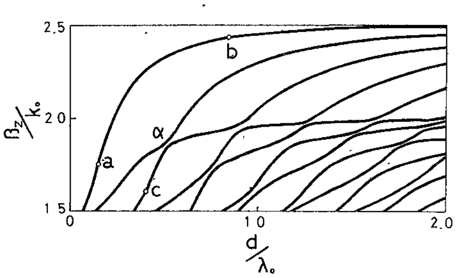

Fig. 4. Normalized propagation constant β_z/k_0 versus normalized frequency d/λ_0 with $\theta = 45°$.

where ξ, η, and ζ are the electrical principal axes of the crystal. In the Cartesian system of coordinates (x,y,z), where the z axis coincides with one of the principal axes ζ, as shown in Fig. 2, the permittivity tensor $\hat{\epsilon}$ becomes

$$\hat{\epsilon} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{xy} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \tag{23}$$

where

$$\epsilon_{xx} = \epsilon_{\xi\xi} \cos^2 \theta + \epsilon_{\eta\eta} \sin^2 \theta$$

$$\epsilon_{xy} = (\epsilon_{\xi\xi} - \epsilon_{\eta\eta}) \sin \theta \cdot \cos \theta$$

$$\epsilon_{yy} = \epsilon_{\eta\eta} \cos^2 \theta + \epsilon_{\xi\xi} \sin^2 \theta$$

$$\epsilon_{zz} = \epsilon_{\zeta\zeta}$$

and θ is an angle made by the x and ξ axes, as shown in Fig. 2. Let us assume further that

$$\epsilon_{\xi\xi} = (2.5)^2 \cdot \epsilon_0 \qquad \epsilon_1 = (1.5)^2 \cdot \epsilon_0$$

$$\epsilon_{\eta\eta} = (2.0)^2 \cdot \epsilon_0 \qquad \epsilon_2 = \epsilon_0$$

$$\epsilon_{\zeta\zeta} = (2.25)^2 \cdot \epsilon_0 \qquad \mu = \mu_0$$

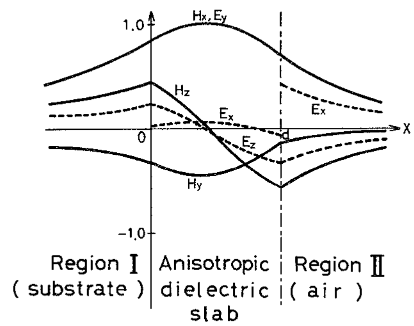

Fig. 5. Electric- and magnetic-field intensity distributions for $d/\lambda_0 = 0.15$, which corresponds to point a in Fig. 4.

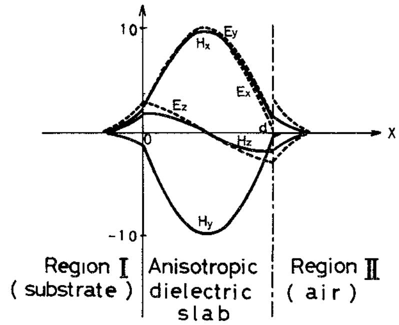

Fig. 6. Electric- and magnetic-field intensity distributions for $d/\lambda_0 = 0.85$, which corresponds to point b in Fig. 4.

where ϵ_0 and μ_0 are the permittivity and permeability of free space, respectively. The calculated values of propagation constant β_z normalized by the propagation constant in free space k_0 $[=\omega(\epsilon_0\mu_0)^{1/2}]$ are shown in Figs. 3 and 4 as a function of the normalized frequency d/λ_0 where d is a thickness of the anisotropic dielectric slab and λ_0 $(=2\pi/k_0)$ is a free-space wavelength. In Fig. 3 it has been assumed that $\theta = 0°$, and in Fig. 4 that $\theta = 45°$.

In the case of $\theta = 0°$ (Fig. 3), the off-diagonal term ϵ_{xy} vanishes, and the permittivity tensor (23) possesses diagonal terms only. In this special case, the TE-modes group and the TM-modes group are separable, and, as we can see from Fig. 3, the normalized propagation constant β_z/k_0 approaches 2.0 and 2.5, respectively, as the normalized frequency d/λ_0 increases (see the Appendix).

On the contrary, if $\theta \neq \pi/2 \cdot n$ $(n = 0,1,2,\cdots)$, the off-diagonal terms in $\hat{\epsilon}$ do not vanish, and the waves are nonseparable into TE and TM modes of waves. That is, the waves are said to be hybrid. The couplings occur between those hybrid modes at particular frequencies at which the propagation constants of the coupled modes become identical. We can see this situation typically at point α, for example, as shown in Fig. 4.

Figs. 5–7 show the calculated electric- and magnetic-field intensity distributions in the guide for various values of d/λ_0, each of which corresponds to the points a, b, and c, respectively, in Fig. 4. With reference to those figures, we recognize both E_z and H_z are finite, namely, the waves are hybrid modes. The field intensities in both

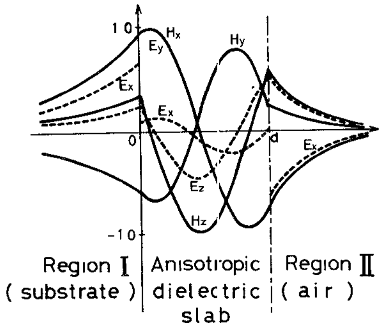

Fig. 7. Electric- and magnetic-field intensity distributions for $d/\lambda_0 = 0.4$, which corresponds to point c in Fig. 4.

substrate material (region I) and the air (region II) decrease rapidly as normalized frequency d/λ_0 increases, and the transmitting power tends to concentrate more and more into the anisotropic dielectric slab waveguide, as would be expected.

Finally, we suggest the possibility of achieving mode discrimination by choosing the value of tensor components in $\hat{\epsilon}$ given by (22) appropriately. In the case of $\theta = 0°$, for instance, the anisotropic dielectric slab waveguide supports only TM modes, provided that the values of tensor components are chosen in such a way that

$$\epsilon_{\xi\xi} > \epsilon_1 > \epsilon_{\eta\eta}, \qquad \theta = 0° \tag{24}$$

where ϵ_1 is a permittivity of the substrate material; whereas only TE modes can be maintained, provided that the condition

$$\epsilon_{\xi\xi} < \epsilon_1 < \epsilon_{\eta\eta}, \qquad \theta = 0° \tag{25}$$

is satisfied. To illustrate the above argument, let us assume that

$$\begin{aligned} \epsilon_{\xi\xi} &= (2.0)^2\cdot\epsilon_0 & \epsilon_1 &= (1.5)^2\cdot\epsilon_0 \\ \epsilon_{\eta\eta} &= (1.25)^2\cdot\epsilon_0 & \epsilon_2 &= \epsilon_0 \\ \epsilon_{\zeta\zeta} &= (1.75)^2\cdot\epsilon_0 & \mu &= \mu_0 \end{aligned}$$

where $\theta = 0°$, which satisfy condition (24). The propagation constant calculated with those parameters is shown in Fig. 8. We can see from Fig. 8 that the TE-modes group is discriminated as expected. It should be pointed out further that the single-mode operation (TM_1 mode, in this example) can be achieved by means of a suitable choice of normalized frequency d/λ_0, as would be seen from Fig. 8. In a similar manner, when $\theta = 90°$, we can eliminate the TM-modes group. This technique for mode discrimination against unwanted modes and the single-mode operation would be useful in integrated optics.

V. Conclusions

The electromagnetic-wave modes propagating in an anisotropic media have been analyzed theoretically, and the propagation conditions under which waves can propagate along the axis of the guide were derived.

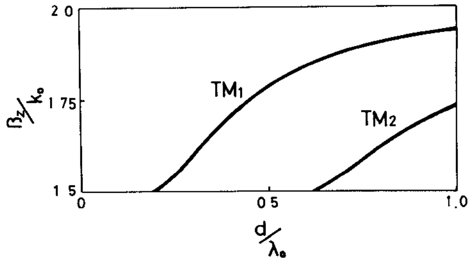

Fig. 8. Normalized propagation constant β_z/k_0 versus normalized frequency d/λ_0 for the anisotropic slab waveguide which supports TM modes only.

A two-dimensional three-layered waveguiding structure consisting of an anisotropic dielectric slab coated on, or embedded in, isotropic surrounding materials has been treated in detail as a typical configuration of the guide. Field-intensity distributions of the propagating modes and their propagation constants have been obtained by numerical computations. Methods for achieving mode discrimination and single-mode operation have been suggested. A class of anisotropic thin-film or slab waveguides using electrooptic or gyrotropic materials seems to be promising for providing an additional variety of schemes in integrated optics such as mode converters, light modulators, optical switches, etc., which are currently subjects of widespread interest.

Appendix

If the permittivity tensor of anisotropic dielectric material possesses diagonal terms only, that is, if $\hat{\epsilon}$ is of the form

$$\hat{\epsilon} = \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \tag{A.1}$$

the propagating modes can be separated into two groups of the modes, the TE-modes group and the TM-modes group.

For the TE-modes group, the field components in an anisotropic dielectric slab are expressed as

$$\begin{aligned} H_z &= A \cos \beta_x x + B \sin \beta_x x \\ E_z &= 0 \\ H_y &= 0 \\ E_y &= -\frac{\omega\mu}{\beta_z} c\{ jA \sin \beta_x x - jB \cos \beta_x x\} \end{aligned} \tag{A.2}$$

where

$$c = \frac{\beta_z}{\beta_x} \qquad \beta_x^2 = k_{yy}^2 - \beta_z^2$$

and the characteristic equation is given by

$$(\alpha_1\alpha_2 - \beta_x^2) \sin \beta_x d + (\alpha_1 + \alpha_2)\beta_x \cdot \cos \beta_x d = 0 \quad \text{(A.3)}$$

where

$$\alpha_1 = (\beta_z^2 - k_1^2)^{1/2} > 0 \qquad \alpha_2 = (\beta_z^2 - k_2^2)^{1/2} > 0.$$

For the TM-modes group, the field components in an anisotropic dielectric slab are expressed as

$$H_z = 0$$

$$E_z = \frac{\omega\mu}{\beta_z} \{A \cos \beta_x x + B \sin \beta_x x\}$$

$$H_y = b\{ jA \sin \beta_x x - jB \cos \beta_x x\}$$

$$E_y = 0 \quad \text{(A.4)}$$

where

$$b = \frac{k_{zz}^2}{\beta_x\beta_z} \qquad \beta_x^2 = k_{zz}^2 - \frac{k_{zz}^2}{k_{xx}^2}\beta_z^2$$

and the characteristic equation is given by

$$\left(\alpha_1\alpha_2 - \frac{k_1^2k_2^2}{k_{zz}^4}\beta_x^2\right) \sin \beta_x d + \left(\frac{k_2^2}{k_{zz}^2}\alpha_1 + \frac{k_1^2}{k_{zz}^2}\alpha_2\right)\beta_x \cdot \cos \beta_x d = 0 \quad \text{(A.5)}$$

where

$$\alpha_1 = (\beta_z^2 - k_1^2)^{1/2} > 0 \qquad \alpha_2 = (\beta_z^2 - k_2^2)^{1/2} > 0.$$

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Electromagnetic-Wave Propagation in the Shielded Ring Line

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Abstract—**A theoretical analysis is presented of a periodic structure consisting of equally spaced perfectly conducting rings. The dispersion relation satisfied by the different modes of the shielded ring line is determined. This analysis shows that cylindrically symmetric modes identical with those of smooth guides and hybrid modes can travel in this periodical structure.**

The asymptotic values of the dispersion relation show the different properties of these hybrid modes. The EH_{n1} modes can be slow, fast, or can travel at light velocity according to the frequency. The EH_{nq} ($q > 1$) modes are fast modes and exchange their cutoff frequencies for particular values of the geometrical parameters of the structure.

These theoretical predictions are verified experimentally by recording the dispersion characteristics of the first modes.

For deflecting radio-frequency structures, the fundamental EH_{11} mode is interesting. This deflection constant is measured on a $\pi/2$ wave structure.

I. Introduction

IN THE SETTING of a research of waveguide structures for RF separators of ultrarelativistic particles, we studied the shielded ring line in which the fundamental hybrid mode is a very interesting deflector mode. In order to study this structure, we followed the same method that Pierce and Field [1] utilized to investigate the propagation of surface waves on the helix. Pierce assumed the helix to be an ideal cylinder with conduction in the helical direction only (the "sheath" helix). The space harmonic fields are then neglected. A more satisfying approach called the "tape" helix was given by Sensiper [2]. He assumed the helix to be wound to an infinitely thin conducting tape and took the electric field at the center line of the tape to be zero. In other respects, he studied a limiting case of the helix: the open ring line composed of equally spaced perfectly conducting rings.

The surface waves which travel along this open ring line are slow waves ($v_p < c$). In the Brillouin diagram $\omega = f(\beta)$ connecting the wave frequency to the phase constant β, the dispersion characteristics of modes are only to be found in the slow-wave domain. If we surround this line with a conducting pipe, the modes can be fast and the dispersion characteristic intersects the straight line $v_p = c$ and is carried on into the fast-wave domain. We have developed the partial study of Falnes [3] on the different modes of this structure.

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